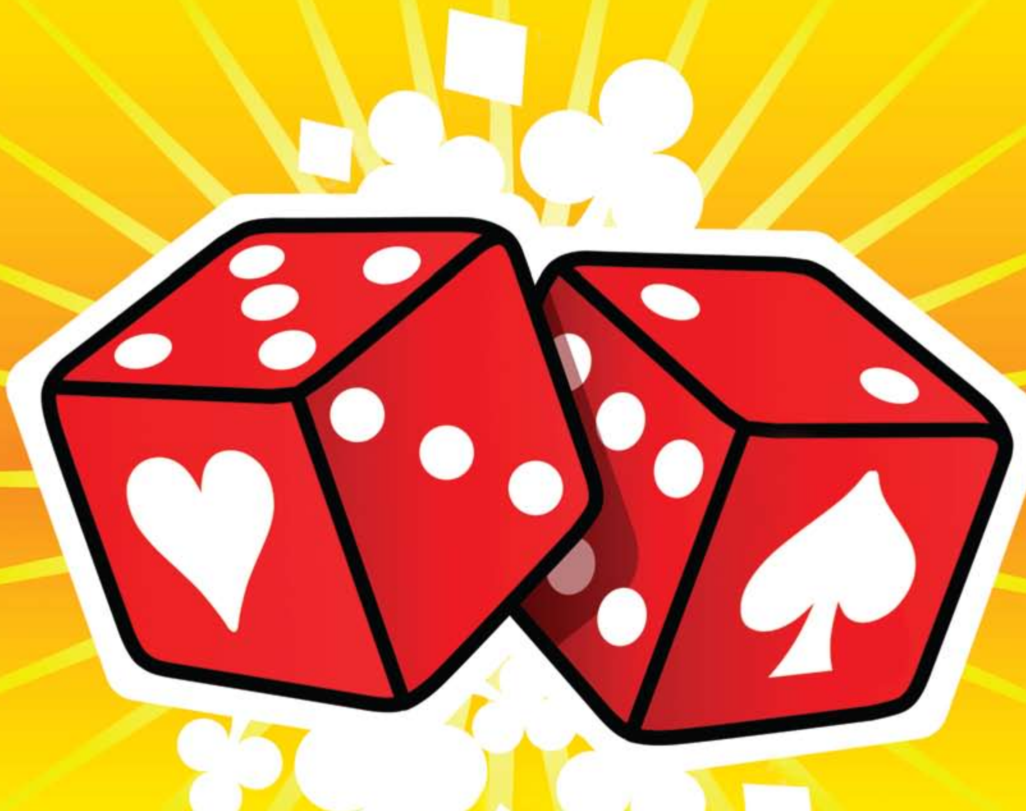
Bridge, Probability & Information



Robert F. MacKinnon

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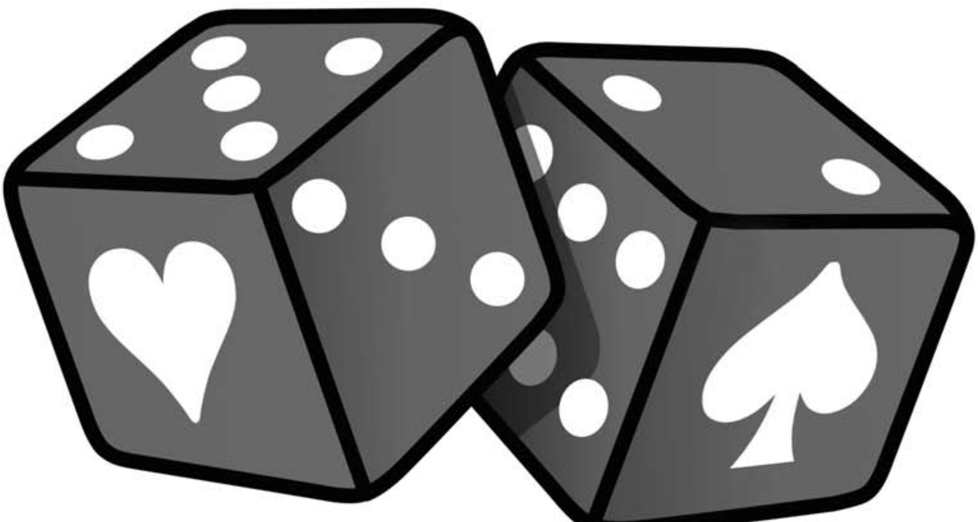

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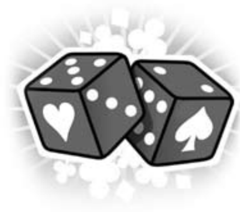
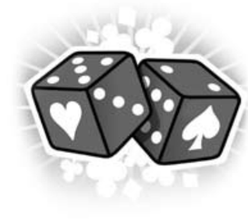


INTRODUCTION



'Begin at the beginning,' the King said, gravely,
'and go on until you come to the end, then stop.'
- from *Alice in Wonderland* by Lewis Carroll (1832-1898)

Perhaps nothing in bridge is as misunderstood as the correct application of Probability to the game. This book represents an attempt to correct some of the worst misconceptions, and at the same time introduce some ideas from the realm of Information Theory, a related branch of mathematics that deals with (among other things) how to make the best guess in the face of partial information. The application of the latter to bridge is self-evident to any moderately experienced player.

What is not self-evident, however, is how to present these mathematical ideas in a way that won't immediately provoke the average reader into closing the book for ever. On the face of it, no advice to a budding author is easier to follow than that given by the King to Alice, for everything must have a beginning, a middle and an end, with the possible exception of time itself. If this were a historical novel, we would be starting in 17th century France where a rich young man, Blaise Pascal, is worried about his gambling debts. The opening scene is set in a Paris tavern.

> *As he sipped his wine, the young man's handsome face became distorted with concern. 'Believe me, Chevalier,' moaned Pascal, 'if I don't find the winning formula soon, my father's vast fortune will disappear into the pockets of unscrupulous gamblers.'*

The subsequent invention of the Theory of Probability could be described in a chapter or two, ending with Pascal's early death in 1662. However, Bridge as we know it wasn't played until 1925, so you see the problem — there would inevitably be dull stretches over the intervening 263 years with only the invention of whist, the precursor of bridge, to lighten the pages.

Whist, bridge without bidding, allowed scientific card play to develop. Some great whist players arose over the centuries: Deschapelles, Yarborough, and Talleyrand are three whose names survive in posterity. The French diplomat made the following famous comment to a colleague: ‘You do not play at whist, *monsieur*? Alas, what a sad old age you are preparing for yourself.’

One of the more familiar stories related to whist comes from a London club frequented by aristocrats. The second Earl of Yarborough had a good understanding of *a priori* probabilities. As he sat down for a game of whist, he would offer to give any player £1000 if during the evening they picked up a hand that contained no card higher than a nine. All he asked was that the player pledge him £1 before each deal. This was a very good proposition for his Lordship as the *a priori* probability of the occurrence of such an event on any given deal is 1826:1.

Back at your author’s dilemma, there are, unfortunately, no good novels revolving around whist playing. It is a pity that Jane Austen didn’t apply herself better to the cause. However, for the purposes of this book, we are free to drift in time. If we assume that the reader knows a lot about Bridge and thinks he knows something about Probability, we can start our work in the middle with the discussion of some bridge deals. In Chapter 1, we shall therefore go over some concepts (such as Restricted Choice) that are familiar, although not necessarily completely understood, with the aim of later describing how these concepts arise from consideration of probabilities. However, if we are to correct wrong impressions, we must sooner or later tackle the basics; that means going back to the beginnings with Pascal, which we do in Chapter 2. Throughout the book, we are going to emphasize that the modern concept of Information is closely linked to Probability. After exploring the application of Information Theory to card play, we shall discuss bidding, a topic that often comes first in bridge books, but here comes at the end.

There are many examples discussed throughout the book, deals played by club players and experts alike. The errors they make are surprisingly similar in nature, which is one lesson the improving player (in which category I place myself, somewhat hopefully) should absorb. Of course, experts make fewer mistakes, but there is a commonality of fallibility that begs to be investigated. When uncertainty is involved, one can’t always make the winning decision. The purpose throughout is to guide players into a way of thinking that allows for continuing improvement. Along the way, the reader will, we hope, discard some misconceptions and learn something of Probability and Information Theory, subjects that have wide application outside the bridge world.

CHAPTER 1

WHEN THE DUMMY COMES DOWN

Whenever you can, count
- Sir Francis Galton (1822-1911), Victorian Scientist

At the core of all sciences are numbers. The central process of bridge playing is counting cards. At the heart of Bridge Probability are ratios of card combinations. That's enough to set us on the right track. Off we go!

When the dummy comes down, a declarer's first duty is to assess his contract, count winners and losers, and begin the process of forming a plan for the play. Next he may scan the cards suit by suit to see which positions need to be tackled early and which guesses should be delayed. Tempting as this approach is to the impatient mind, it is the wrong approach — an essential first step in the counting process has been missed.

Not many bridge books will tell you this, but even before considering the implications of the opening lead, declarer should count the number of cards held *jointly* with dummy in each suit. If between his hand and the dummy he finds eight spades, then the defenders must hold the remaining five; if seven hearts, the defenders hold six, and so on. This is known as **counting the sides**. As play progresses, changes occur, but the division of 'sides' is the firm framework within which such changes occur, as cards can't jump from one suit to another.

Bridge players are very familiar with individual hand patterns. They recognize as 'normal' a 4432 shape which occurs for 21.5% of the hands, and as 'flat' a 4333 shape, which occurs 10.5% of the time. The divisions of sides have corresponding patterns. An 8765 pattern is the most common, occurring in 23.6% of deals, and next is the 7766 pattern, which occurs in 10.5% of deals. The former is considered 'normal', the latter 'flat', requiring special treatment.

Let's consider a 7-7-6-6 pattern first as it occurs in the defenders' side. (The hyphen signs indicate that the numbers relate to the suits in strict order, thus seven spades, seven hearts, six diamonds and six clubs.)

I	II	III
♠ 4 - 3	♠ 3 - 4	♠ 5 - 2
♥ 3 - 4	♥ 4 - 3	♥ 1 - 6
♦ 3 - 3	♦ 3 - 3	♦ 4 - 2
♣ 3 - 3	♣ 3 - 3	♣ 3 - 3

In the absence of bidding, Conditions I and II are the two most likely distributions of the suits among the defenders. The strings of numbers represent the double helix of the deal's composition. They determine whether you have encountered something ordinary or something more unusual, like the one shown in Condition III. If the bidding has gone 1NT-3NT and the lead is a low spade, declarer should assume Condition I as a working hypothesis, and if the lead is a low heart, Condition II. Begin with what is most likely and work from there, keeping in mind the Scottish proverb 'What may be may not be'.

I	II	III
♠ 4 - 4	♠ 4 - 4	♠ 4 - 4
♥ 3 - 4	♥ 4 - 3	♥ 3 - 4
♦ 3 - 3	♦ 3 - 3	♦ 4 - 2
♣ 3 - 2	♣ 2 - 3	♣ 2 - 3

With 8-7-6-5 as the defensive sides, both 4333 and 4432 are common hand patterns. On a low spade lead, Condition I is more likely than Condition II, because in the latter case, a heart might have been led. Under Condition III, a diamond might have been led, but it is normal to lead a major against a 3NT contract.

Eventually, these observations could determine in what manner declarer plays the club suit. This is not much to go on, we agree, but it is what's available. The next step is to gather more information at minimum cost, since the information gathered may alter one's estimate of the splits within a suit.

This concept of counting sides will be treated in greater detail later in the book, especially in Chapter 5, but for now the time has come to provide some sustenance in the form of examples of how the process works. If you should find these no more than demonstrations of common sense, then you have captured the essence of Probability.

Counting Cards - Alice in Bridgeland

'Can you do Addition?' the White Queen said. 'What are one and one and one and one and one and one and one and one and one and one?'
'I don't know', said Alice, 'I lost count.'
'She can't do Addition', the Red Queen interrupted.
-from *Through the Looking Glass* by Lewis Carroll (1832-1898)

Behind this abstract, one can imagine a kindly, middle-aged Professor Charles Dodgson attempting to teach little Alice how to count out the trumps during the play of a hand of whist. Not easy for a young person just introduced to a complex game, and not, incidentally, the best approach at bridge where one gets to see the cards in the dummy. No, the best way is to introduce a pattern made up of your cards plus the dummy's cards, the division of sides, and to count all four suits at once by modifying the pattern as new cards appear during the play. This is a much easier way to keep track than by counting cards one by one, suit by suit, which overtaxes the memory.

There is a further advantage to this approach, besides ease of calculation, which is that it gets a declarer to look at the deal as a whole. The play in one suit may be affected by the distribution of cards in another suit, as we shall demonstrate with two deals played in 6NT, one where a complete count can be obtained and a second where an inferential count is used. (Yes, we know you could count the hand in the time-honored way too: we are just trying to show you the 'sides' process.) Both deals involve playing the combination of ♦KQ9 opposite ♦A1074. The correct play in the suit depends on the conditions in the outside suits at the time of decision. It is not true that finesses are destined to fail half the time.

Here is a deal played by Alice later in life, her golden ringlets now a tarnished silver.

Dealer South
NS Vulnerable

Edward
♠ A J 2
♥ K Q 4
♦ K Q 9
♣ K 6 4 2

Freddy
♠ K Q 9 8 6 5
♥ 9 8 3
♦ J 5 3
♣ 9

Rose
♠ 7 3
♥ 10 6 5 2
♦ 8 4 2
♣ J 10 8 3

Alice
♠ 10 4
♥ A J 7
♦ A 10 7 6
♣ A Q 7 5

Freddy	Edward	Rose	Alice
			1NT
2♠	6NT	all pass	

Aunt Alice is hosting her weekly game at home with her feckless nephew, Freddy, and his spouse. Now a grandmother, she is still quite capable of opening a strong 1NT with less than the required strength (in those days, 16 HCP was considered the absolute minimum). Freddy, who is showing signs of restlessness, tries to upset her with a silly overcall, but her ever-trusting husband gives her a sporting raise. Freddy leads a straightforward ♠K and awaits developments with a stifled yawn.

'I thought the clubs might split badly,' says Edward apologetically as he puts down the dummy. It is always wise to cover yourself with Alice.

'Thank you, Edward. Your values are quite suitable,' replies his mate reassuringly.

This looks like an easy twelve tricks: two spades, three hearts, three diamonds and four clubs on the expected 3-2 split. The defenders' cards are most likely divided as shown under Condition I below.

	I	II	III	IV
♠	6 - 2	6 - 2	6 - 2	6 - 2
♥	3 - 4	2 - 5	2 - 5	3 - 4
♦	2 - 4	3 - 3	4 - 2	3 - 3
♣	2 - 3	2 - 3	1 - 4	1 - 4

Still, Edward may be right, and if the clubs don't split 3-2, Alice will need to make four tricks in the diamond suit. This can be done in three ways: finessing in either direction or playing for the jack to come down in three rounds. It is all a matter of counting and planning ahead for each eventuality.

Alice takes the ♠A and returns the suit, Freddy taking his ♠Q and exiting safely with the ♠6, a card signifying nothing. His long-suffering partner discards the ♥2. This looks like it might be a count card from a five-card suit. In that case, the distribution of sides might be that listed under Condition II. There is still no problem as long as clubs behave, but when Alice cashes the ♣KQ, leaving the ♣A in dummy for the purposes of transportation, Freddy discards the ♠8. Now one must consider the possibility of the distribution under Condition III where the percentage play is to finesse Freddy for the ♦J.

Thanks to her foresight in not releasing the ♣A in her hand, Alice is able to play off the top hearts to confirm the expected presence of a doubleton heart in the West hand, but to her mild surprise, the hearts split evenly, so she obtains the full count represented by Condition IV. The hand has become an open book and Alice plays off the top diamonds for twelve tricks.

'Jolly well done, Aunt Alice,' says Freddy. 'I don't really see how you figured out to drop my ♦J. Against the odds, but your only chance, I imagine.'

'It was largely a matter of Luck,' says Alice, graciously ringing for brandy and chocolate biscuits. With his wife present, it was not the time to advise her nephew on the need to count out a hand and draw the obvious conclusion.

The Inferential Count

We may not be able to get certainty, but we can get probability,
and half a loaf is better than no bread.
- C.S.Lewis (1898-1963)

Dealer South
NS Vulnerable

Joel
♠ A 10 2
♥ K Q 4 3
♦ K Q 9
♣ K 10 4

Sam
♠ K Q 9 8 6 3
♥ 9 7
♦ 8 5
♣ Q 9 5

Joan
♠ 7 5
♥ 10 8 6 5 2
♦ J 4 3 2
♣ J 6

Alice
♠ J 4
♥ A J
♦ A 10 7 6
♣ A 8 7 3 2

Sam	Joel	Joan	Alice
			1♣
2♠	3♠	pass	4♦
pass	4♠[1]	pass	4NT[2]
pass	6NT	all pass	

1. RKCB for diamonds.
2. Three keycards.

Let's jump ahead to London in the swinging sixties. In 1964, Great Britain has won the Women's Team Bridge Olympiad and hopes are high that the men can rise from their third-place finish and regain their former supremacy in next year's world championships. The great-grandchildren of Alice and the rest are playing for high stakes in a Mayfair club. The new Alice, a smashing boy-cut blonde, is affectionately known as 'Mousetrap' because of her penchant for sharp penalty doubles when lesser beings stray. Sitting West is a rich real estate developer from New York. The bidding systems are ever-changing, and sometimes misunderstandings arise, but good contracts are often reached nonetheless. Due to an accident of good fortune arising from a hazy recollection of the latest craze from Italy, Alice gets to play in a 6NT contract with a decision to be made on the same diamond holding. Although West's overcall is now part of a system and not a mere flight of fancy, it nonetheless still proves ineffective and the lead is the same ♠K.

'Sorry if I got this wrong,' says gentlemanly Joel as he lays down his excellent dummy.

Alice wins the ♠A in dummy and plays off the ♥AJ before establishing a second spade trick. Sam exits with the ♠6 as East discards the ♥2. A group of admirers who include Maurice Harrison-Gray lean forward in their chairs to see if this petite blonde can make twelve tricks where the Losing Trick Count predicts just the obvious eleven. The top hearts are cashed; when West shows up with six spades and two hearts and East with two spades and five hearts, the remaining manageable possibilities include:

	I	II
♠	6 - 2	6 - 2
♥	2 - 5	2 - 5
♦	3 - 3	2 - 4
♣	2 - 3	3 - 2

Alice needs to get the diamonds right, but she can't get a full count because her clubs are not sufficiently robust. However, an inferential count is available using the Principle of Restricted Choice, which has become all the rage after Terence Reese showed everyone how it works. Playing off the ♣A and ♣K, she arrives at this four-card ending:

	Dummy	
	♠ — ♥ — ♦ K Q 9 ♣ 10	
Sam ♠ 9 ♥ — ♦ 8 5 ♣ Q		**Joan** ♠ — ♥ — ♦ J 4 3 2 ♣ —
	Alice ♠ — ♥ — ♦ A 10 7 6 ♣ —	

With no one having thrown a diamond yet, it's clear neither defender started with four diamonds and the ♣Q, but there is another inference to be drawn. On the second round of clubs, Joan played the ♣J. The Principle of Restricted Choice tells us that Sam is twice as likely to hold the missing queen as is Joan. Thus Joan most probably began with two spades, five hearts, four diamonds and just two clubs (Condition II). The diamond finesse through East is a 4:2 favorite, and so it transpires.

'Well done, my dear,' whispers Harrison-Gray as Sam makes some notes in a little black book. 'Let me put your name before the selection committee for Buenos Aires. We pre-war ancients need rejuvenation if we are to uphold Britannia's honor.'

'Young lady, you should take up Maury on his offer,' advises the New Yorker. 'The US of A will have Mrs Hayden on the Open Team, who you very much remind me of. She plays as well as any man.'

'And better than most,' sniffs Joan.

'You're so sweet, Maurice and Sam, and I do so love the tango...' smiles Alice. 'Help me here, Joel. You be the judge: can women today consider themselves the equal of men?'

'I myself rate them above men, always have,' replies Joel drily, 'although I consider it largely a matter of personal preference.'

Percentage Play

Principle and fact are like eyes and feet.
- Zen Master Fayan Wenyi (885-958)

> JAYNES' PRINCIPLE: In making inferences on the basis of partial information we must use the probability distribution which has maximum uncertainty subject to whatever is known.

The above statement is one of the outstanding achievements of the 20th century with regard to the application of probability theory to scientific endeavor. The consequences to playing a bridge deal are easily stated. Whether we think of maximum uncertainty or ratios of card combinations, it comes to the same thing: play for splits that are as even as possible under the circumstances — there are more cases, and therefore they are more likely. Of course, circumstances may change dramatically when the defenders are forced to make a revealing play, either by showing out of a suit, or playing a card that damages their chances (Restricted Choice).

Of course, the more information at one's disposal, the better will be the basis for a decision and the more likely the even split, so the process feeds on itself.

ADVANCED

TAKE THE GUESSWORK OUT OF BRIDGE

Bridge, unlike chess, is a game of incomplete information. We bid with thirteen cards in view, and play each deal seeing only twenty-six: the positions of the rest we must deduce from the auction and from the cards played. Bridge players deal in likelihoods, and only rarely in certainties, therefore some knowledge of the laws of probability is a critical weapon in a successful player's arsenal.

In this book, using a semi-fictional narrative approach, the author develops the ideas (but not the equations!) that underlie basic probability and its modern descendant, information theory, and shows how both fields relate to bridge. In fact, they have enormous practical application to the game. Among the topics discussed:

- The idea of visualizing 'sides', the complete combined holdings of both defenders, and not just the splits in individual suits
- How a known split in one suit can affect the odds in another
- Empirical rules to help make decisions based on incomplete information or in a situation too complex to analyze accurately
- How *a priori* probabilities (the ones with which we are all familiar) change with each card played
- How an imbalance of vacant places in the defenders' hands affects the odds — and when to change your line of play as a result
- The 'Monty Hall Problem' and its bridge cousin, Restricted Choice
- HCP distribution — what partner's bidding tells you about where his high cards are
- Information versus frequency: the trade-off in choosing conventions
- Losing Trick Count — does it work, and if so, why?
- Probability, statistics and the Law of Total Tricks — how far can you rely on the LAW?
- Cost versus gain: information theory as applied to bidding systems
- Using statistics to help you choose a bidding system that works for you

ROBERT F. MacKINNON lives in Victoria, Canada. His bridge writings include a blog on mathematical issues in bridge, various magazine articles, and two books of bridge fiction, including the remarkable *Samurai Bridge*.

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